IGCSE Mathematics 0580 Extended

Complete Revision Notes with Free PDF Download

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What this PDF covers. Every Extended topic on the Cambridge IGCSE Mathematics 0580 syllabus for first examination 2025 through to 2027, written for students revising on their own. Worked examples are checked line by line. Formulas are set out as proper fractions so nothing is ambiguous when you copy them down.

How to use it. Read a topic, cover the worked example, redo it on paper, then compare. The red boxes are the mistakes examiners see most often. The amber boxes are marks you can pick up cheaply.

Exam structure, Extended tier

Extended candidates take Paper 2 and Paper 4. Each paper is worth 50 percent of the final grade. Extended candidates are eligible for grades A* to E.

Paper	Calculator	Time	Marks	Weighting
Paper 2 (Extended)	Not allowed	2 hours	100	50 percent
Paper 4 (Extended)	Scientific calculator required	2 hours	100	50 percent

COMMON MISTAKES

- Paper 2 is the non-calculator paper. This was introduced with the 2025 syllabus, so any resource still describing Paper 2 as a short 70 mark paper is out of date.
- Both papers contain structured and unstructured questions.
- Do not confuse these with the Core papers. Paper 1 and Paper 3 are Core, 1 hour 30 minutes and 80 marks each. Mixing them up is the single most common error in free revision material.

The nine topics

	Topic		Topic
1	Number	6	Trigonometry
2	Algebra and graphs	7	Transformations and vectors
3	Coordinate geometry	8	Probability
4	Geometry	9	Statistics
5	Mensuration		

Topic 1

Number

1.1 Types of number

Type	Definition	Examples
Natural numbers	The counting numbers, starting at 1.	1, 2, 3, 4, 5, ...
Integers	All whole numbers, including negatives and zero.	..., −2, −1, 0, 1, 2, ...
Rational numbers	Any number that can be written as $\frac{p}{q}$ where p and q are integers and $q \neq 0$.	$\frac{3}{4}$, 0.5, −2, 0.333...
Irrational numbers	Cannot be written as a fraction. The decimal never terminates and never recurs.	$\pi = 3.14159...$, $\sqrt{2} = 1.41421...$
Real numbers	All rational and irrational numbers together.	Every point on the number line
Prime numbers	Greater than 1, with exactly two factors: 1 and itself.	2, 3, 5, 7, 11, 13, 17, 19, 23
Square numbers	An integer multiplied by itself.	1, 4, 9, 16, 25, 36, 49, 64, 81, 100
Cube numbers	An integer multiplied by itself three times.	1, 8, 27, 64, 125, 216

COMMON MISTAKES

- 1 is **not** a prime number. It has only one factor. The smallest prime is 2.
- 2 is the only even prime number.
- 0 is not a natural number in Cambridge usage. It is an integer.
- $\sqrt{2}$ is irrational, but $\sqrt{4} = 2$ is rational. A root is not automatically irrational.

1.2 Standard form

KEY FACTS

A number in standard form is written as $A \times 10^n$, where $1 \leq A < 10$ and n is an integer.

- Large numbers give a positive index: $32\,000 = 3.2 \times 10^4$
- Small numbers give a negative index: $0.005\,67 = 5.67 \times 10^{-3}$
- Multiplying: multiply the front numbers, add the indices.
- Dividing: divide the front numbers, subtract the indices.

WORKED EXAMPLE

$(2.4 \times 10^3) \times (3 \times 10^2) = (2.4 \times 3) \times 10^{3+2} = 7.2 \times 10^5$

$(6 \times 10^8) \div (2 \times 10^3) = \frac{6}{2} \times 10^{8-3} = 3 \times 10^5$

Check: $7.2 \times 10^5 = 720\,000$, and $2400 \times 300 = 720\,000$. Correct.

1.3 Percentages

KEY FACTS

- Increase by r percent: multiply by $(1 + \frac{r}{100})$. A price of \$200 raised by 15 percent becomes $200 \times 1.15 = \$230$.
- Decrease by r percent: multiply by $(1 - \frac{r}{100})$. A price of \$80 cut by 20 percent becomes $80 \times 0.80 = \$64$.
- Percentage change $= \frac{\text{new value} - \text{original value}}{\text{original value}} \times 100$
- Reverse percentage: if 85 percent of x is 340, then $x = \frac{340}{0.85} = 400$.
- Simple interest: $I = \frac{P \times R \times T}{100}$, where P is the principal, R the rate per year and T the time in years.
- Compound interest: $A = P(1 + \frac{r}{100})^n$, where A is the final amount and n the number of years.

WORKED EXAMPLE

\$500 is invested at 4 percent compound interest per year for 3 years.

$A = 500 \times (1.04)^3 = 500 \times 1.124\,864 = \$562.43$ (2 dp)

Check: $1.04 \times 1.04 \times 1.04 = 1.124\,864$. Correct.

EXAM TIP

- Reverse percentage is not the same as taking the percentage off again. If a price rises by 20 percent to \$60, the original is $\frac{60}{1.2} = \$50$, not $60 - 12 = \$48$.

1.4 Indices and roots

Rule	Worked example
$a^m \times a^n = a^{m+n}$	$3^4 \times 3^2 = 3^6 = 729$
$a^m \div a^n = a^{m-n}$	$5^6 \div 5^2 = 5^4 = 625$
$(a^m)^n = a^{mn}$	$(2^3)^4 = 2^{12} = 4096$
$a^0 = 1$, for $a \neq 0$	$7^0 = 1$
$a^{-n} = \frac{1}{a^n}$	$2^{-3} = \frac{1}{8} = 0.125$
$a^{\frac{1}{n}} = \sqrt[n]{a}$	$27^{\frac{1}{3}} = \sqrt[3]{27} = 3$
$a^{\frac{m}{n}} = (\sqrt[n]{a})^m$	$16^{\frac{3}{4}} = (\sqrt[4]{16})^3 = 2^3 = 8$
$a^{-\frac{m}{n}} = \frac{1}{a^{\frac{m}{n}}}$	$8^{-\frac{2}{3}} = \frac{1}{(\sqrt[3]{8})^2} = \frac{1}{4} = 0.25$

EXAM TIP

- Work the root first, then the power. It keeps the numbers small: $16^{\frac{3}{4}} = (\sqrt[4]{16})^3 = 2^3 = 8$. Doing it the other way round means $\sqrt[4]{(16^3)} = \sqrt[4]{4096}$, which is far harder without a calculator.

1.5 Surds

KEY FACTS

- $\sqrt{a} \times \sqrt{b} = \sqrt{(ab)}$ and $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{(\frac{a}{b})}$
- Simplify by pulling out square factors: $\sqrt{50} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$
- Collect like surds: $3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$. You cannot add $\sqrt{2}$ and $\sqrt{3}$.
- Rationalise a denominator by multiplying top and bottom by the surd: $\frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$
- For a two term denominator, multiply by the conjugate: $\frac{1}{3 + \sqrt{2}} = \frac{3 - \sqrt{2}}{(3 + \sqrt{2})(3 - \sqrt{2})} = \frac{3 - \sqrt{2}}{7}$

EXAM TIP

- Paper 2 has no calculator, so exact surd answers are expected. Leaving $5\sqrt{2}$ as 7.07 can lose the accuracy mark.

1.6 Upper and lower bounds

KEY FACTS

A value rounded to the nearest unit u lies within $\frac{u}{2}$ of the stated value.

- A length of 24 cm to the nearest cm has bounds $23.5 \le L < 24.5$
- Adding: add the lower bounds for the lower bound, add the upper bounds for the upper bound.
- Subtracting: lower bound = (lower of first) − (upper of second).
- Multiplying: multiply lower with lower, and upper with upper.
- Dividing: lower bound = $\frac{\text{lower of first}}{\text{upper of second}}$, upper bound = $\frac{\text{upper of first}}{\text{lower of second}}$.

WORKED EXAMPLE

A car travels 240 m (to the nearest 10 m) in 12.0 s (to the nearest 0.1 s). Find the upper bound of the speed.

Distance bounds: $235 \le d < 245$. Time bounds: $11.95 \le t < 12.05$

Upper bound of speed = $\frac{245}{11.95}$ = 20.5 m/s (3 sf)

Check: the largest distance over the shortest time gives the fastest speed. Correct.

1.7 Ratio, proportion and rate

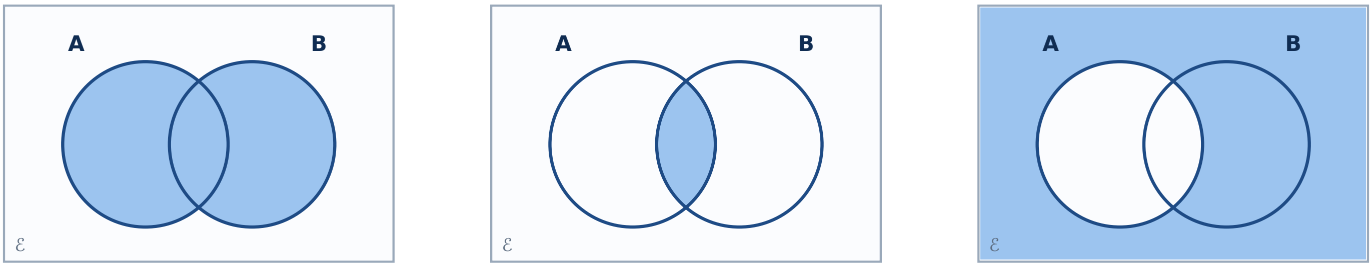
KEY FACTS

- Simplify a ratio by dividing both parts by the HCF: 12 : 18 = 2 : 3
- Share 480 in the ratio 3 : 5. Total parts = 8, so one share is $480 \times \frac{3}{8} = 180$ and the other is $480 \times \frac{5}{8} = 300$.
- Direct proportion: $y = kx$. Double x and y doubles. If y is proportional to x^2, then $y = kx^2$.

- Inverse proportion: $y = \frac{k}{x}$. Double x and y halves.
- Speed $= \frac{\text{distance}}{\text{time}}$, distance = speed × time, time $= \frac{\text{distance}}{\text{speed}}$
- Density $= \frac{\text{mass}}{\text{volume}}$. Pressure $= \frac{\text{force}}{\text{area}}$.
- 1 km = 1000 m. 1 hour = 60 minutes = 3600 seconds. To convert km/h to m/s, multiply by $\frac{1000}{3600}$, which is the same as dividing by 3.6.

1.8 Sets and Venn diagrams

Notation	Meaning
$\mathcal{E}$	The universal set, everything under consideration
$A \cup B$	Union. Everything in A or in B or in both.
$A \cap B$	Intersection. Only what is in both A and B.
A'	Complement. Everything in $\mathcal{E}$ that is not in A.
$n(A)$	The number of elements in A
$x \in A$	x is an element of A
$A \subseteq B$	A is a subset of B
$\varnothing$	The empty set, containing no elements



A ∪ B
Union: in A, in B, or in both

A ∩ B
Intersection: in both A and B

A′
Complement: everything not in A

The universal set is the rectangle. Shade the region the question asks for before you count.

EXAM TIP

- Fill a Venn diagram from the middle outwards. Put $n(A \cap B)$ in first, then subtract it from each total before writing the outer regions.

Topic 2

Algebra and Graphs

2.1 Expanding and factorising

KEY FACTS

- $a(b + c) = ab + ac$
- $(x + a)(x + b) = x^2 + (a + b)x + ab$
- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $(a + b)(a - b) = a^2 - b^2$, the difference of two squares
- Take out the HCF: $6x^2 + 9x = 3x(2x + 3)$
- $x^2 + 5x + 6 = (x + 2)(x + 3)$. Find two numbers that add to 5 and multiply to 6.
- Factorise in pairs: $2x^2 + 3x - 2 = (2x - 1)(x + 2)$

WORKED EXAMPLE

Expand and simplify $(2x + 1)^2$

$= 4x^2 + 4x + 1$

Check at $x = 3$: left side $= 7^2 = 49$. Right side $= 36 + 12 + 1 = 49$. Correct.

Factorise $x^2 - 5x + 6$

Two numbers adding to -5 and multiplying to 6 are -2 and -3, so the answer is $(x - 2)(x - 3)$.

Check at $x = 4$: left side $= 16 - 20 + 6 = 2$. Right side $= 2 \times 1 = 2$. Correct.

2.2 Algebraic fractions

KEY FACTS

- Simplify by factorising top and bottom, then cancel common brackets: $\frac{x^2 - 9}{x^2 + 7x + 12} = \frac{(x + 3)(x - 3)}{(x + 3)(x + 4)} = \frac{x - 3}{x + 4}$
- Add or subtract using a common denominator: $\frac{2}{x} + \frac{3}{x + 1} = \frac{2(x + 1) + 3x}{x(x + 1)} = \frac{5x + 2}{x(x + 1)}$
- Multiply: multiply the numerators and multiply the denominators, then cancel.
- Divide: turn the second fraction upside down and multiply.

COMMON MISTAKES

- You can only cancel a factor of the whole numerator with a factor of the whole denominator. In $\frac{x + 4}{4}$ the 4s do not cancel.

2.3 Solving equations

KEY FACTS

- Linear: do the same thing to both sides. $3x - 7 = 14$ gives $3x = 21$, so $x = 7$.
- Simultaneous by elimination: match one pair of coefficients, then add or subtract.
- Simultaneous by substitution: rearrange one equation, put it into the other.
- Quadratic formula, for $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

- Discriminant $b^2 - 4ac$. If it is positive there are two solutions, if it is zero there is one repeated solution, if it is negative there are no real solutions.
- Inequalities: solve as for equations, but reverse the sign whenever you multiply or divide by a negative number.

WORKED EXAMPLE

Solve $x^2 - 5x + 6 = 0$, so $a = 1$, $b = -5$, $c = 6$

$b^2 - 4ac = 25 - 24 = 1$, so $x = \frac{5 \pm 1}{2}$

$x = 3$ or $x = 2$

Check: substituting $x = 3$ gives $9 - 15 + 6 = 0$. Substituting $x = 2$ gives $4 - 10 + 6 = 0$. Both correct.

Solve $2x + y = 7$ and $x - y = 2$ together.

Adding the two equations removes y: $3x = 9$, so $x = 3$, and then $y = 1$.

Check: $2(3) + 1 = 7$ and $3 - 1 = 2$. Both correct.

2.4 Completing the square

KEY FACTS

Write $x^2 + bx + c$ in the form $(x + p)^2 + q$, where $p = \frac{b}{2}$ and $q = c - p^2$.

- $x^2 + 6x + 1 = (x + 3)^2 - 9 + 1 = (x + 3)^2 - 8$
- The turning point of $y = (x + p)^2 + q$ is at $(-p, q)$. Here it is $(-3, -8)$.
- If there is a coefficient in front of x^2, take it out first: $2x^2 + 8x + 5 = 2(x^2 + 4x) + 5 = 2(x + 2)^2 - 3$
- This form also solves the equation: $(x + 3)^2 = 8$ gives $x = -3 \pm 2\sqrt{2}$.

2.5 Sequences

KEY FACTS

- Arithmetic: a constant common difference d. The nth term is $a + (n - 1)d$. For 3, 7, 11, 15 the first term is 3 and $d = 4$, so the nth term is $4n - 1$.
- Geometric: each term is multiplied by a common ratio r. The nth term is $a \times r^{n-1}$. For 2, 6, 18, 54 the nth term is $2 \times 3^{n-1}$.
- Quadratic: the second difference is constant. If the second difference is D, the nth term starts with $\frac{D}{2}n^2$.
- Fibonacci style: each term is the sum of the previous two. 1, 1, 2, 3, 5, 8, 13, 21

WORKED EXAMPLE

Find the nth term of 4, 7, 12, 19, 28

First differences: 3, 5, 7, 9. Second difference: 2, so the sequence is quadratic.

$\frac{D}{2} = \frac{2}{2} = 1$, so the nth term begins with n^2.

Subtract n^2 from each term: $4 - 1 = 3$, $7 - 4 = 3$, $12 - 9 = 3$, $19 - 16 = 3$. The remainder is a constant 3.

nth term = $n^2 + 3$

Check at $n = 5$: $25 + 3 = 28$. Correct.

2.6 Functions

KEY FACTS

- $f(x)$ means the rule f applied to x. To evaluate $f(3)$, replace every x with 3.
- Composite function $fg(x)$ means do g first, then f. Work from the inside out.
- Inverse function $f^{-1}(x)$ undoes f. Write $y = f(x)$, rearrange to make x the subject, then swap the letters.
- The domain is the set of inputs allowed. The range is the set of outputs produced.

WORKED EXAMPLE

$f(x) = 2x + 3$ and $g(x) = x^2$

$fg(2)$: first $g(2) = 4$, then $f(4) = 11$

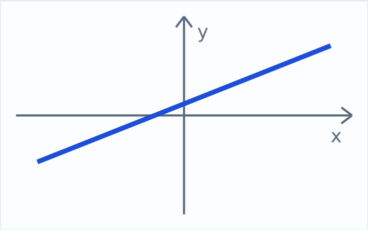
$gf(2)$: first $f(2) = 7$, then $g(7) = 49$. So fg and gf are different.

Inverse of f: let $y = 2x + 3$, so $x = \frac{y-3}{2}$, giving $f^{-1}(x) = \frac{x-3}{2}$

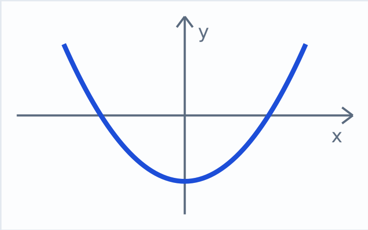
Check: $f(5) = 13$ and $f^{-1}(13) = \frac{10}{2} = 5$. Correct.

2.7 Graphs of functions

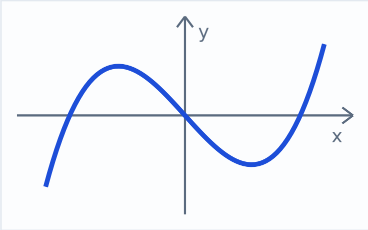
Type	General form	Shape and key features
Linear	$y = mx + c$	Straight line. m is the gradient, c is the y intercept.
Quadratic	$y = ax^2 + bx + c$	Parabola. If $a > 0$ it opens upwards, if $a < 0$ it opens downwards. It is symmetrical about $x = -\frac{b}{2a}$.
Cubic	$y = ax^3 + bx^2 + cx + d$	Two turning points, or none. Opposite ends go in opposite directions.
Reciprocal	$y = \frac{k}{x}$	Hyperbola in two separate parts, with the axes as asymptotes. It never touches either axis.
Exponential	$y = a^x$	Always above the x axis. Rises steeply if $a > 1$, falls if $0 < a < 1$. It passes through (0, 1).
Trigonometric	$y = \sin x$, $y = \cos x$, $y = \tan x$	Repeating waves. sin and cos repeat every 360°, tan every 180°.



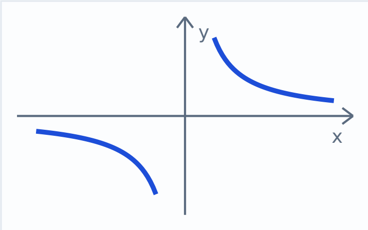
Linear
$y = mx + c$



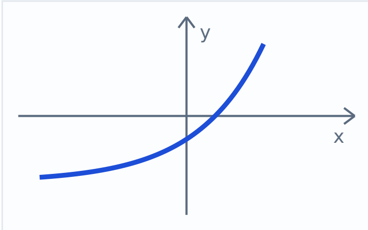
Quadratic
$y = ax^2 + bx + c$



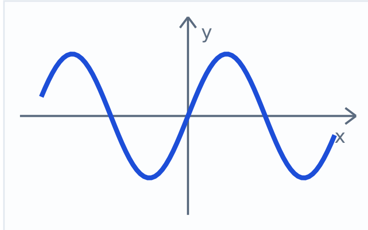
Cubic
$y = ax^3 + bx^2 + cx + d$



Reciprocal
y = k ÷ x



Exponential
$y = a^x$

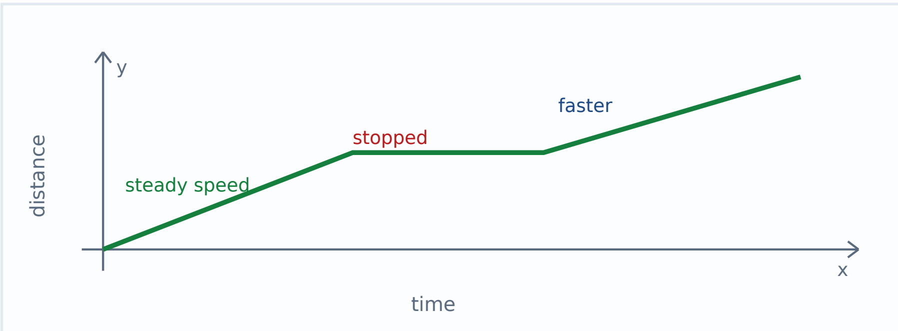


Trigonometric
y = sin x

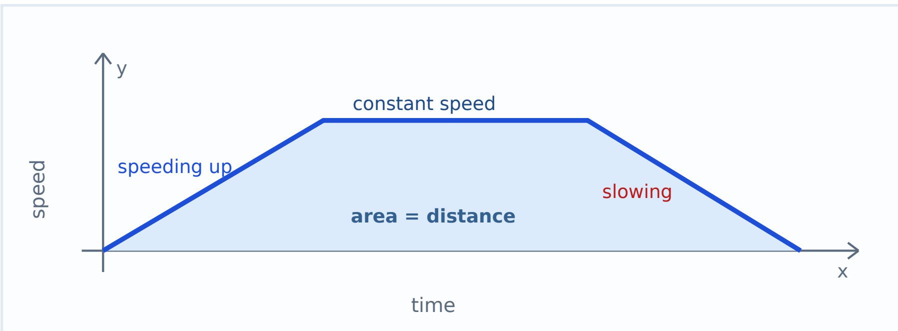
Learn the shape first. If your sketch has the wrong shape, no amount of correct plotting rescues it.

KEY FACTS

- Gradient = $\frac{y_2 - y_1}{x_2 - x_1}$, that is the change in y divided by the change in x.
- Straight line: $y = mx + c$, or $y - y_1 = m(x - x_1)$
- Parallel lines have equal gradients.
- Perpendicular lines: if one gradient is m, the other is $-\frac{1}{m}$. The two gradients multiply to −1.
- Turning point of $y = ax^2 + bx + c$ is at $x = -\frac{b}{2a}$. Substitute to find y.
- On a distance time graph the gradient is the speed, and a horizontal line means stopped.
- On a speed time graph the gradient is the acceleration, and the area underneath is the distance travelled.



Distance time graph
Gradient = speed. A flat line means stopped.



Speed time graph
Gradient = acceleration. Area underneath = distance.

On a distance time graph the gradient is the speed. On a speed time graph the area underneath is the distance travelled.

2.8 Differentiation

KEY FACTS

If $y = ax^n$, then $\frac{dy}{dx} = anx^{n-1}$. Differentiate each term separately. A constant term differentiates to zero.

- $\frac{dy}{dx}$ gives the gradient of the curve at any value of x.
- At a turning point the gradient is zero, so solve $\frac{dy}{dx} = 0$.
- To decide which kind of turning point it is, test the gradient just before and just after. Negative then positive is a minimum. Positive then negative is a maximum.

WORKED EXAMPLE

Find the turning point of $y = x^2 - 6x + 5$

$\frac{dy}{dx} = 2x - 6$

Setting $2x - 6 = 0$ gives $x = 3$, and then $y = 9 - 18 + 5 = -4$.

The turning point is $(3, -4)$. At $x = 2$ the gradient is -2 and at $x = 4$ it is $+2$, so it is a minimum.

Check: completing the square gives $y = (x - 3)^2 - 4$, which has its minimum at $(3, -4)$. The two methods agree.

2.9 Exponential growth and decay

KEY FACTS

Final amount = initial amount × $(\text{multiplier})^n$, where n is the number of time periods.

- Growth of r percent per period: multiplier $= 1 + \frac{r}{100}$
- Decay of r percent per period: multiplier $= 1 - \frac{r}{100}$
- A population of 5000 growing at 3 percent a year reaches $5000 \times 1.03^{10} = 6720$ after 10 years.
- A car worth \$18 000 losing 15 percent a year is worth $18\,000 \times 0.85^4 = \$9396$ after 4 years.

Topic 3

Coordinate Geometry

KEY FACTS

- Distance between (x_1, y_1) and $(x_2, y_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
- Gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Line through (x_1, y_1) with gradient m: $y - y_1 = m(x - x_1)$
- For the x intercept set $y = 0$. For the y intercept set $x = 0$.
- Perpendicular bisector of AB: use the midpoint of AB and the gradient $-\frac{1}{\text{m of AB}}$.

WORKED EXAMPLE

Distance from A(1, 2) to B(5, 5)

$= \sqrt{(5 - 1)^2 + (5 - 2)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$

Check: the sides are 3 and 4, a 3, 4, 5 triangle. Correct.

Line through (2, 3) and (6, 11)

Gradient $= \frac{11 - 3}{6 - 2} = \frac{8}{4} = 2$

$y - 3 = 2(x - 2)$, which simplifies to $y = 2x - 1$

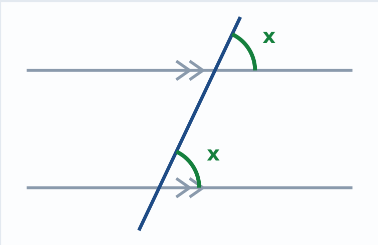
Check at $x = 6$: $y = 12 - 1 = 11$. Correct.

Topic 4

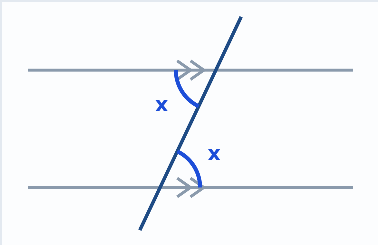
Geometry

4.1 Angle facts

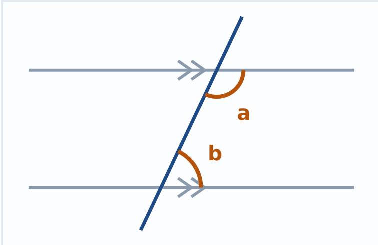
Fact	Rule
Angles on a straight line	Add up to 180°
Angles at a point	Add up to 360°
Vertically opposite angles	Are equal
Angles in a triangle	Add up to 180°
Angles in a quadrilateral	Add up to 360°
Corresponding angles	Equal, in the F shape between parallel lines
Alternate angles	Equal, in the Z shape between parallel lines
Co-interior angles	Add up to 180°, in the C shape between parallel lines
Exterior angle of a triangle	Equals the sum of the two opposite interior angles
Sum of interior angles of an n sided polygon	$(n - 2) \times 180°$
Interior angle of a regular n sided polygon	$\frac{(n - 2) \times 180°}{n}$
Exterior angle of a regular n sided polygon	$\frac{360°}{n}$



Corresponding
F shape, the angles are equal
both angles marked x are equal



Alternate
Z shape, the angles are equal
both angles marked x are equal

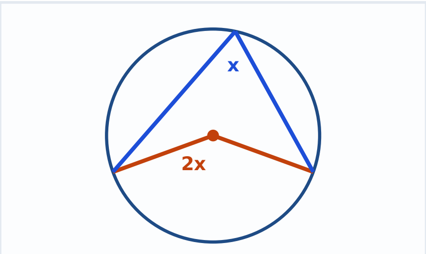


Co-interior
C shape, the angles add to 180°
a + b = 180°

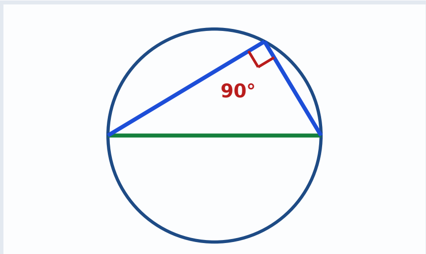
Trace the shape with your finger. The arcs mark the angles the rule is talking about.

4.2 Circle theorems

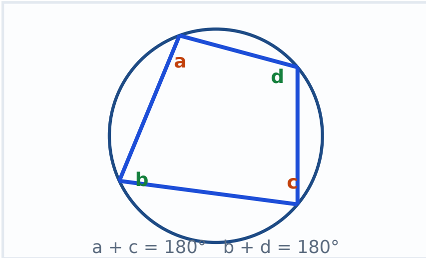
Theorem	Statement
Angle at the centre	The angle at the centre is twice the angle at the circumference, standing on the same arc.
Angle in a semicircle	The angle in a semicircle is always 90°.
Angles in the same segment	Angles standing on the same chord, in the same segment, are equal.
Cyclic quadrilateral	Opposite angles add up to 180°.
Tangent and radius	A tangent meets the radius at 90° at the point of contact.
Two tangents	Two tangents drawn from the same external point are equal in length.
Alternate segment	The angle between a tangent and a chord equals the angle in the alternate segment.



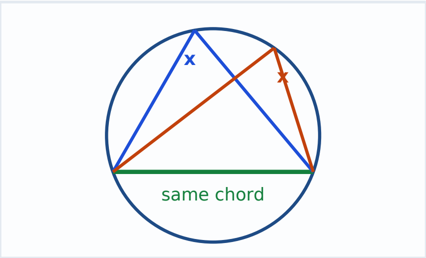
Angle at the centre
Centre angle = 2 × circumference angle



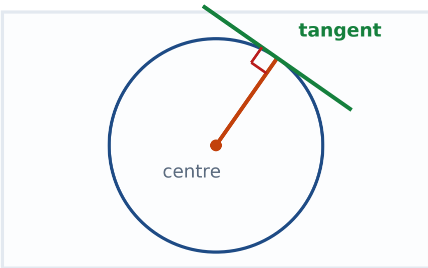
Angle in a semicircle
The angle on the diameter is 90°



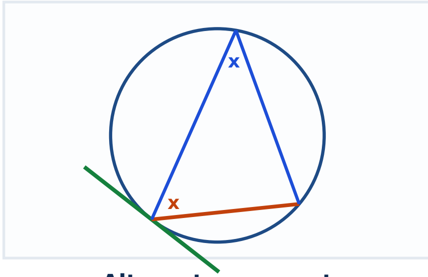
Cyclic quadrilateral
Opposite angles add to 180°



Angles in the same segment
Equal, when on the same side of the chord



Tangent and radius
They meet at 90° at the point of contact



Alternate segment
Tangent chord angle = angle in the other segment

COMMON MISTAKES

- Name the theorem in your working. Cambridge awards a mark for the reason, so an unexplained correct answer can still drop a mark.
- Angles in the same segment are only equal when they are on the same side of the chord.
- The alternate segment theorem only applies where a tangent touches the circle and a chord leaves from that same point.

Topic 5

Mensuration

Shape	Area or volume	Perimeter or surface area
Rectangle	$A = l \times w$	$P = 2(l + w)$
Triangle	$A = \frac{1}{2} \times \text{base} \times \text{height}$	P = sum of the three sides
Parallelogram	$A = \text{base} \times \text{height}$	$P = 2(a + b)$
Trapezium	$A = \frac{1}{2}(a + b) \times h$, where a and b are the parallel sides	P = sum of the four sides
Circle	$A = \pi r^2$	$C = 2\pi r$
Sector	$A = \frac{\theta}{360} \times \pi r^2$	$\text{Arc} = \frac{\theta}{360} \times 2\pi r$
Cuboid	$V = l \times w \times h$	$SA = 2(lw + lh + wh)$
Prism	V = area of cross section × length	SA = 2 × cross section + perimeter × length
Cylinder	$V = \pi r^2 h$	Curved SA = $2\pi rh$. Total SA = $2\pi r^2 + 2\pi rh$
Cone	$V = \frac{1}{3}\pi r^2 h$	Slant height $l = \sqrt{(r^2 + h^2)}$. Curved SA = πrl. Total SA = $\pi rl + \pi r^2$
Sphere	$V = \frac{4}{3}\pi r^3$	$SA = 4\pi r^2$
Pyramid	$V = \frac{1}{3} \times \text{base area} \times \text{height}$	SA = base area + the triangular faces

WORKED EXAMPLE

A cylinder has radius 3 cm and height 10 cm.

$V = \pi \times 3^2 \times 10 = 90\pi = 282.7\ \text{cm}^3$ (1 dp)

Total SA $= 2\pi(3)^2 + 2\pi(3)(10) = 18\pi + 60\pi = 78\pi = 245.0\ \text{cm}^2$ (1 dp)

Check: $90 \times 3.14159 = 282.74$ and $78 \times 3.14159 = 245.04$. Correct.

EXAM TIP

- Check the units. Area is cm^2, volume is cm^3. Writing the wrong unit loses a mark even when the number is right.
- For the slant height of a cone use Pythagoras with r and h. Do not use the slant height in the volume formula, and do not use the vertical height in the curved surface area formula.

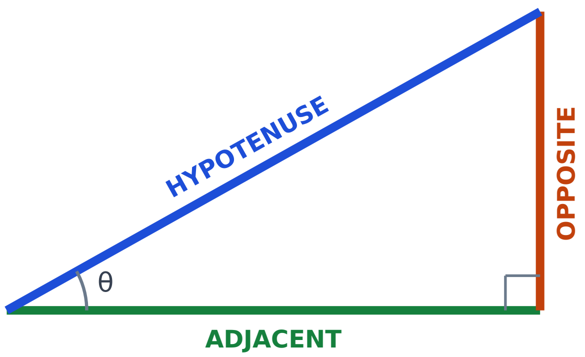
Topic 6

Trigonometry

6.1 Right angled triangles, SOHCAHTOA

Each side is colour coded below, and the same colours are used in the triangle. Match the colour, not the letter, and the three ratios stop looking alike.

SOH	CAH	TOA
$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$	$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$	$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$



The hypotenuse is always opposite the right angle. Opposite and adjacent depend on where θ is.

KEY FACTS

- Hypotenuse: the longest side, always opposite the right angle. It never changes.
- Opposite: the side facing the angle θ you are working with.
- Adjacent: the side next to θ that is not the hypotenuse.
- To find a side, rearrange. For example opposite = hypotenuse × $\sin\theta$.
- To find an angle, use the inverse. For example $\theta = \tan^{-1}(\frac{3}{4}) = 36.9°$ (1 dp).

- Pythagoras: $a^2 + b^2 = c^2$, where c is the hypotenuse.

COMMON MISTAKES

- Opposite and adjacent swap over if you move to the other angle in the same triangle. Label the sides again for every new angle.
- Make sure the calculator is in degrees mode, not radians.

6.2 Exact values to learn for Paper 2

θ	sinθ	cosθ	tanθ
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
45°	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	undefined

6.3 Triangles without a right angle

KEY FACTS

- Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. Use it when you know two angles and a side, or two sides and an angle that is not between them.
- Cosine rule for a side: $a^2 = b^2 + c^2 - 2bc \cdot \cos A$. Use it when you know two sides and the angle between them.
- Cosine rule for an angle: $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$. Use it when you know all three sides.
- Area of any triangle $= \frac{1}{2}ab \cdot \sin C$, where C is the angle between sides a and b.
- Bearings are measured clockwise from north and written with three figures. North is 000°, east is 090°, south is 180°, west is 270°.
- In three dimensions, find a right angled triangle inside the solid first, then apply Pythagoras or SOHCAHTOA to that triangle.

EXAM TIP

- Side a must be opposite angle A. Getting the pairing wrong is the usual reason a sine rule answer comes out impossible.

Topic 7

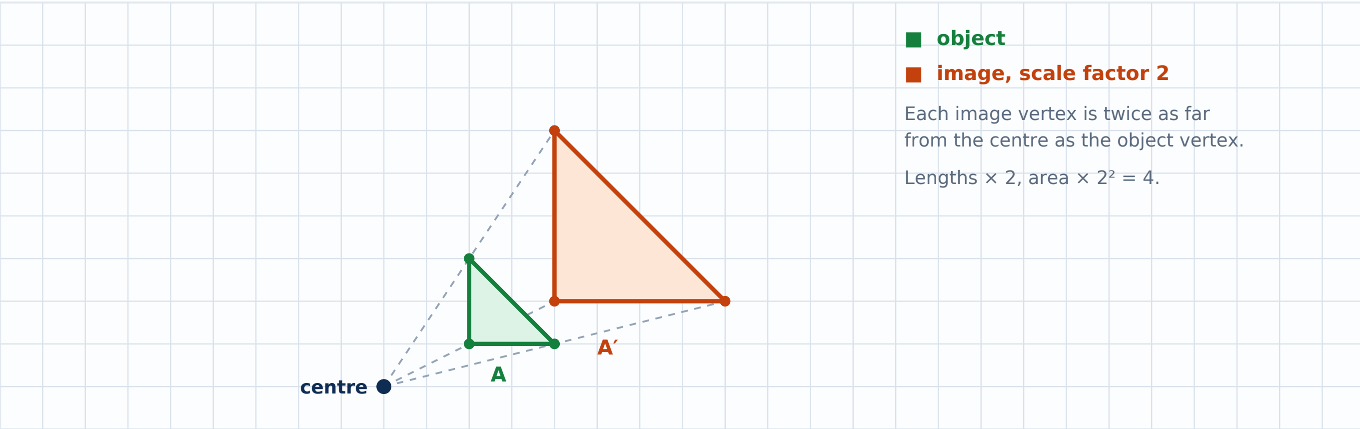
Transformations and Vectors

7.1 Transformations

Transformation	What you must state to describe it fully
Translation	The column vector, giving the movement across and up or down
Reflection	The equation of the mirror line
Rotation	The centre of rotation, the angle, and the direction
Enlargement	The centre of enlargement and the scale factor

KEY FACTS

- Scale factor = $\frac{\text{image length}}{\text{object length}}$
- A scale factor above 1 enlarges. Between 0 and 1 it makes the shape smaller. A negative scale factor puts the image on the other side of the centre, turned upside down.
- Area scale factor = (length scale factor)^2
- Volume scale factor = (length scale factor)^3
- Congruent shapes are the same shape and the same size.
- Similar shapes are the same shape but a different size.



Measure from the centre of enlargement. Every image distance is the scale factor times the object distance.

7.2 Vectors

KEY FACTS

- A vector has both size and direction. Write it as a column vector or as a bold letter.
- Adding: add the components. If a = (3, 2) and b = (1, −4) then a + b = (4, −2).
- Subtracting: subtract the components. a − b = (3 − 1, 2 − (−4)) = (2, 6).
- Multiplying by a number: 3a = (9, 6).
- Magnitude of (x, y) = $\sqrt{(x^2 + y^2)}$

- Two vectors are parallel when one is a multiple of the other.
- Midpoint M of AB has position vector OA + $\frac{1}{2}$AB.

Topic 8

Probability

KEY FACTS

- P(event) = $\frac{\text{number of favourable outcomes}}{\text{total number of equally likely outcomes}}$
- Probability is always between 0 (impossible) and 1 (certain).
- P(not A) = 1 − P(A)
- Mutually exclusive events: P(A or B) = P(A) + P(B)
- Independent events: P(A and B) = P(A) × P(B)
- Conditional probability: P(A given B) = $\frac{\text{P(A and B)}}{\text{P(B)}}$
- Tree diagrams: multiply along the branches, add between the branches.
- Relative frequency = $\frac{\text{number of times the event happened}}{\text{total number of trials}}$

WORKED EXAMPLE

A bag holds 3 red balls and 5 blue balls. Two are taken out without replacement.

P(both red) = $\frac{3}{8} \times \frac{2}{7} = \frac{6}{56} = \frac{3}{28}$

P(both blue) = $\frac{5}{8} \times \frac{4}{7} = \frac{20}{56} = \frac{5}{14}$

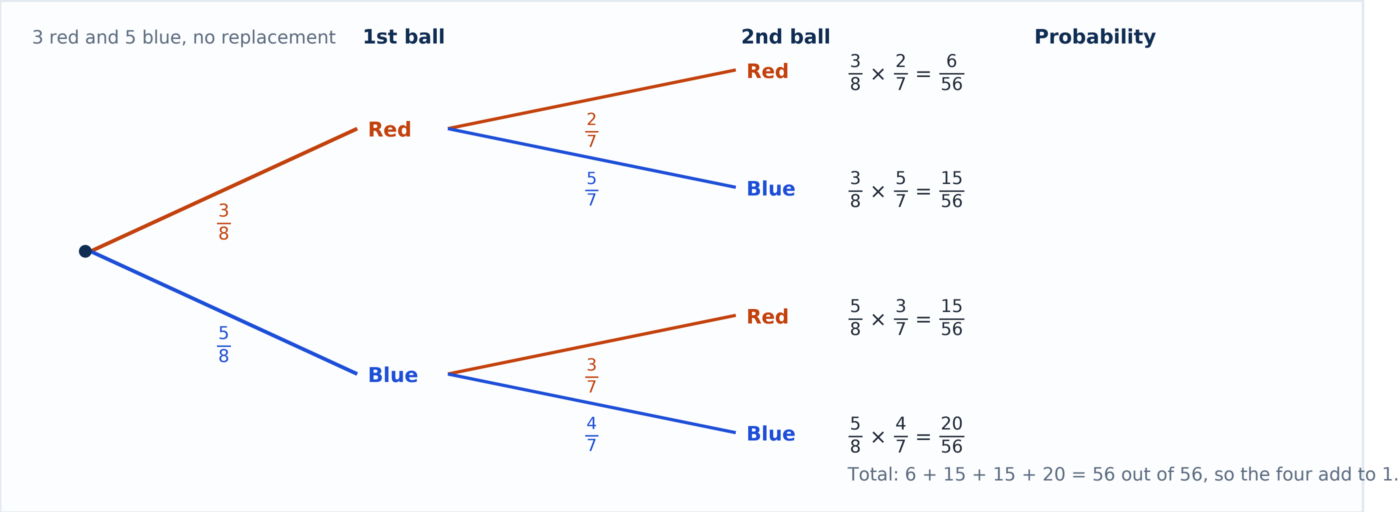
P(same colour) = $\frac{3}{28} + \frac{10}{28} = \frac{13}{28}$

Check: P(different colours) = $2 \times \frac{3}{8} \times \frac{5}{7} = \frac{15}{28}$, and $\frac{13}{28} + \frac{15}{28} = 1$. Correct.

COMMON MISTAKES

- Without replacement means the second denominator drops by one. Forgetting this is the most common error in the whole topic.

8.1 Tree diagrams



Multiply along the branches. Add between the branches. The four end results always total 1.

Topic 9

Statistics

9.1 Averages and spread

Measure	Definition	For 3, 5, 7, 7, 8, 9, 12
Mean	The total of all the values divided by how many there are	$\frac{51}{7} = 7.29$ (2 dp)
Median	The middle value once the data is in order	7, the 4th of 7 values
Mode	The value that occurs most often	7, which appears twice
Range	Largest value minus smallest value	$12 - 3 = 9$
Interquartile range	$Q_3 - Q_1$, the spread of the middle half	$Q_1 = 5$ and $Q_3 = 9$, so IQR = 4

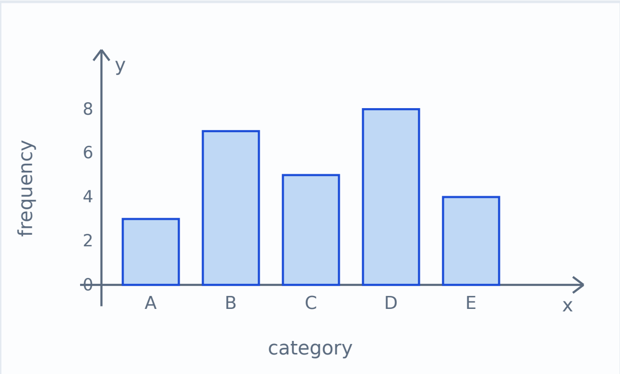
KEY FACTS

- Estimated mean from grouped data = $\frac{\Sigma(\text{midpoint} \times \text{frequency})}{\Sigma\text{frequency}}$
- The median is affected least by an extreme value, so it is the fairer average for skewed data.
- The interquartile range ignores the extremes, so it is a more reliable measure of spread than the range.

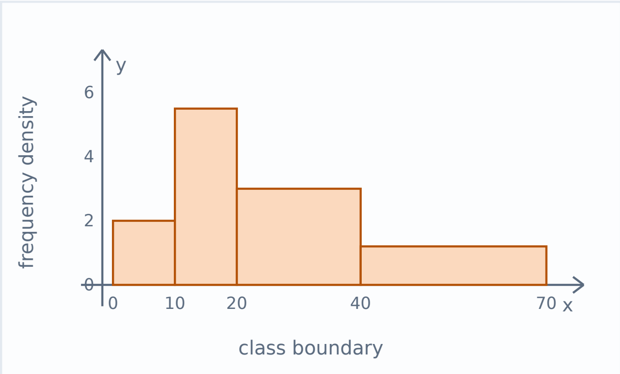
9.2 Statistical diagrams

Diagram	Used for	Points to remember
Bar chart	Discrete or categorical data	Equal width bars with gaps between them
Pie chart	Showing proportions	Angle = $\frac{\text{frequency}}{\text{total}} \times 360°$

Diagram	Used for	Points to remember
Histogram	Continuous grouped data	No gaps. Height = frequency density = $\frac{\text{frequency}}{\text{class width}}$
Frequency polygon	Comparing distributions	Plot the class midpoints against frequency
Cumulative frequency	Median, quartiles and percentiles	Plot the upper class boundary against the running total. Mark points with a cross and join with a smooth curve.
Box plot	Spread and median at a glance	Shows the minimum, Q_1, median, Q_3 and maximum
Scatter diagram	Correlation between two variables	Draw the line of best fit through the mean point

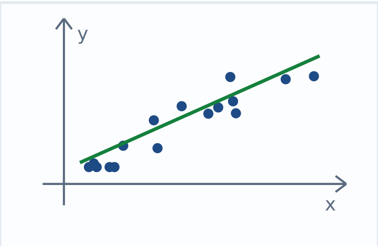


Bar chart, discrete data
Equal widths, gaps between the bars

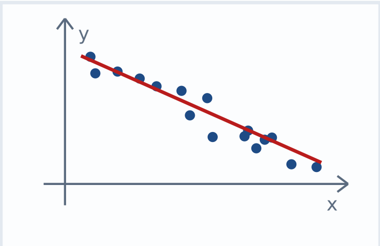


Histogram, continuous data
Unequal widths allowed, no gaps

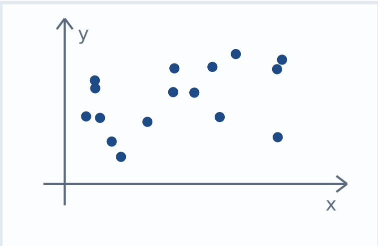
A bar chart has gaps and its height is frequency. A histogram has no gaps and its height is frequency density.



Positive correlation
as x rises, y rises

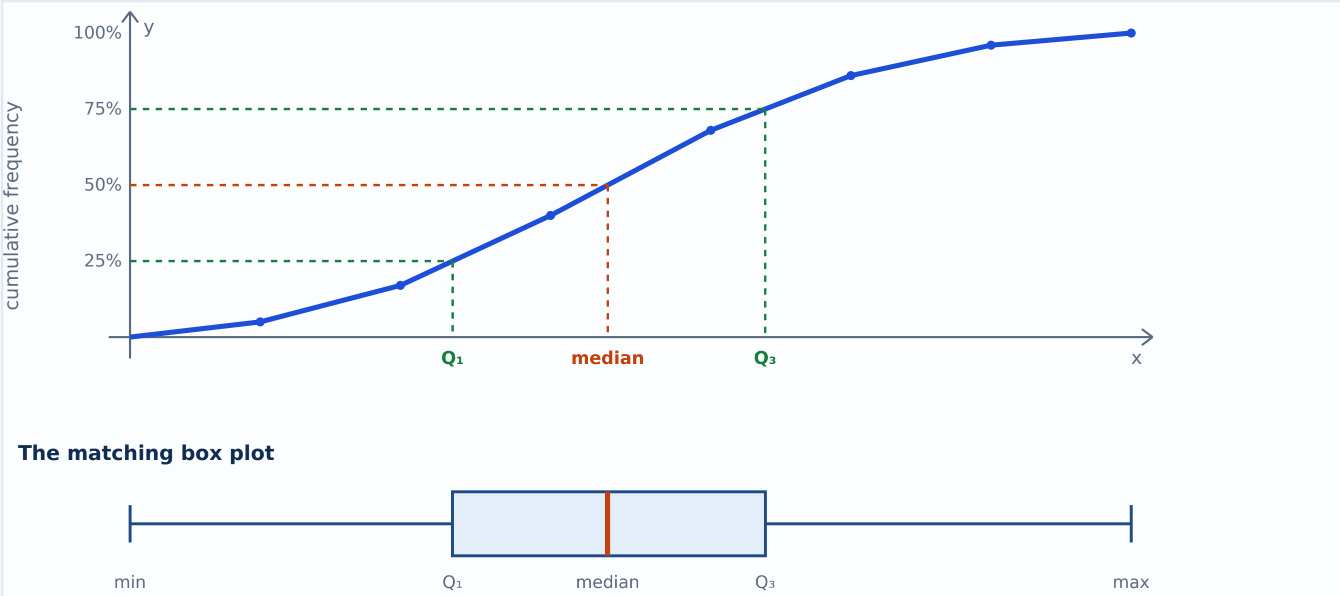


Negative correlation
as x rises, y falls



No correlation
no line of best fit

Correlation is not the same as cause. Never draw the line of best fit beyond the data you have.



IQR = $Q_3 - Q_1$, the width of the box

Read across from the cumulative frequency axis, then straight down to the value on the horizontal axis.

KEY FACTS

- Positive correlation: as one variable increases, so does the other.
- Negative correlation: as one increases, the other decreases.
- No correlation: the points show no pattern.
- Correlation is not the same as cause. Two things can move together without one causing the other.
- Do not read values beyond the range of the data. The pattern is not reliable out there.

COMMON MISTAKES

- On a histogram the bar height is frequency density, never frequency. To recover the frequency, multiply the height by the class width.

Final Revision Checklist

Tick each line only when you can do it without looking at your notes.

☐ Write a number in standard form and calculate with it
☐ Work out reverse percentages and compound interest
☐ Apply all the index laws, including negative and fractional powers
☐ Simplify surds and rationalise a denominator
☐ Find upper and lower bounds and use them in a calculation
☐ Use set notation and complete a Venn diagram
☐ Expand, factorise and simplify algebraic fractions
☐ Solve linear, simultaneous and quadratic equations
☐ Complete the square and read off the turning point
☐ Find the nth term of a linear, quadratic and geometric sequence
☐ Work with composite and inverse functions
☐ Recognise and sketch every graph type on the syllabus

- ☐ Differentiate a polynomial and find its turning points
- ☐ Use the distance, midpoint and gradient formulas
- ☐ State and apply every circle theorem by name
- ☐ Use every mensuration formula, with the correct units
- ☐ Apply SOHCAHTOA, the sine rule and the cosine rule
- ☐ Describe every transformation fully
- ☐ Add, subtract and scale vectors, and find a magnitude
- ☐ Complete a tree diagram, with and without replacement
- ☐ Draw and read a histogram and a cumulative frequency curve